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NON-STATIONARY INFLUENCE FUNCTION FOR AN UNBOUNDED ANISOTROPIC KIRCHHOFF-LOVE SHELL

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The purpose of this article is to investigate the process of the influence of a nonstationary load on an arbitrary region of an elastic anisotropic cylindrical shell. The approach to the study of the propagation of forced transient oscillations in the shell is based on the method of the influence function, which represents normal displacements in response to the action of a single load concentrated along the coordinates. For the mathematical description of the instantaneous concentrated load, the Dirac delta functions are used. To construct the influence function, expansions in exponential Fourier series and integral Laplace and Fourier transforms are applied to the original differential equations. The original integral Laplace transform is found analytically, and for the inverse integral Fourier transform, a numerical method for integrating rapidly oscillating functions is used. The convergence of the result in the Chebyshev norm is estimated. The practical significance of the work is that the obtained results can be used by scientists or students to solve new problems of dynamics of cylindrical shells on an elastic basis under pulse loads. The found non-stationary influence function opens up possibilities for studying the stress-strain state, solving nonstationary inverse and contact problems for anisotropic shells, studying nonstationary dynamics in the case of nonzero initial conditions, and also when constructing integral equations of the boundary element method.

Key words: dynamics, influence function, generalised functions, integral transformations

INTRODUCTION

In many areas of technology, for example, in rocket and missile engineering, aircraft industry, mechanical engineering and construction, such structural element as shells is widely used. The continuous increase in the level and dynamics of improvement and development of new promising designs entails the imposition of higher requirements for knowledge of vibration propagation patterns in shells. A special place is occupied by the analysis of the propagation of non-stationary oscillations, due to the fact that in such problems the variability of the required solution is substantially inhomogeneous in time and coordinates. The stress-strain behaviour of cylindrical shells under the influence of shock loads simulated by impulse functions is of theoretical and applied interest.

Problems devoted to the study of the unsteady dynamics of isotropic elastic Kirchhoff-Love plates have been studied most fully in the book by A.G. Gorshkov, A.L. Medvedsky, L.N. Rabinsky, and D.V. Tarlakovsky [1]. In the works of A.E. Bogdanovich [2, 3], the author studied a wide range of problems in the dynamics of orthotropic cylindrical shells, their axisymmetric and non-axisymmetric deformation during longitudinal impact. In addition, non-axisymmetric deformation at unsteady external pressure was considered. Much attention is paid to the derivation and analysis of nonlinear equations of motion for orthotropic shells, the study of the applicability of the Kirchhoff-Love model in problems of dynamics. Methods for solving geometrically nonlinear problems of the dynamics of imperfect cylindrical shells are presented. On their basis, formulation and development of methods for analysing the strength of cylindrical shells made of laminated composites under dynamic compressive loads.

The work of T.B. Koshkina [4] is devoted to the problem of deformation of reinforced cylindrical shells under the action of dynamic compressive loads. The paper considers the basic equations of the nonlinear theory of laminated orthotropic cylindrical shells, supported by stiffeners, and solved non-axisymmetric problems of dynamic buckling of imperfect orthotropic cylindrical shells with stiffening ring using the Bubnov-Galerkin method based on numerous approximations of displacements. Methods were developed for solving non-axisymmetric problems of deformation of orthotropic cylindrical shells, reinforced by annular or longitudinal enforcement ribs.

Issues of the influence of longitudinal-radial vibrations on a viscoelastic cylindrical shell were reflected in [5]. Approximate equations of longitudinal vibrations of a cylindrical shell were formulated and their area of application was determined. The approach to the study was based on the consideration of a cylinder as a three-dimensional deformable body, solving three-dimensional equations of the dynamics of this body by applying the integral Fourier and Laplace transforms, constructing general solutions of boundary value problems in transformations, expanding the stress-strain state of a body in terms of the degree of the radial coordinate, determining the required functions from three-dimensional boundary conditions

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for given external unsteady forces and stresses.

The papers [6-8] reflect the application of the method of influence functions in solving non-stationary problems of the theory of elasticity and the theory of shells. Non-stationary contact problems for thin cylindrical, spherical shells and elastic half-space are investigated [9, 10]. The direct and inverse problems for a Timoshenko-type beam of finite length under the action of an unsteady load, issues related to the identification of defects in an elastic rod are considered in [11-13]. The case of transient action of a rigid indenter on an elastic half-plane is considered [14, 15]. In papers [16-18] the issues of unsteady dynamics and the peculiarities of constructing the influence function for anisotropic plates and shells are considered. The approach to the solution is based on the method of Green's functions and the principle of superposition, according to which the required solution is related to the load by means of an integral operator of convolution type in spatial variables and in time [19-21]. The core of this operator is the influence function for the plate, which represents the normal displacements in response to the action of a single coordinate and time-concentrated load. Dirac delta functions are used for the mathematical description of this load. Integral Laplace and Fourier transforms are used to construct the influence function.

The problems of unsteady dynamics of elastic anisotropic shells are insufficiently studied at the moment. In the present paper, the process of the influence of a nonstationary load on an arbitrary region of an elastic anisotropic cylindrical shell is analytically investigated. For the first time, presumably, a non-stationary influence function for an anisotropic shell was constructed, which opens up the possibility of extensive applied and scientific research. Its application is possible in the study of the stress-strain state, the solution of unsteady inverse and contact problems for anisotropic shells, as well as the study of unsteady dynamics in the case of nonzero initial conditions.

MATERIALS AND METHODS

An unbounded thin elastic anisotropic cylindrical Kirchhoff-Love shell of constant thickness *h*, radius *R* and material density *ρ* is considered (Fig. 1). The authors consider the anisotropy of the material such that the elastic medium has one surface of symmetry. In this case, this surface is the middle surface of the shell. In this case, the material under consideration is characterised by six independent elastic constants [22]: $C^{1111}=c_{12}$; $C^{1122}=c_{12}$; $C^{1112}=c_{16}$; $C^{2222}=c_{22}$; $C^{1222}=c_{26}$; $C^{1212}=c_{66}$.

The problem is solved in the cylindrical coordinate system *ORαz* associated with the *z*-axis of the cylindrical shell. At the initial moment of time, the shell is acted upon by an unsteady normal pressure *P(α, z, τ)*, distributed over an arbitrary region *D* belonging to the lateral surface of the shell [23-25].

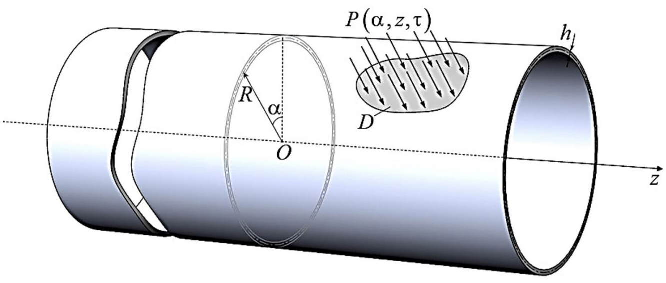


Figure 1: Cylindrical shell under unsteady pressure

The statement of the problem includes the equations of motion of the Kirchhoff-Love elastic shell, the corresponding geometric and physical relations, taking into account the properties of the anisotropy of the material of the shell under study [26, 27]. The displacement equations of motion and differential operators have the form (Eqs. 1-12):

$$\frac{\partial^2 u_\alpha}{\partial \tau^2} = K_{11}(u_\alpha) + K_{12}(u_z) + K_{13}(w) + q_\alpha \qquad (1)$$

$$\frac{\partial^2 u_z}{\partial \tau^2} = K_{21}(u_\alpha) + K_{22}(u_z) + K_{23}(w) + q_z \qquad (2)$$

$$\frac{\partial^2 w}{\partial \tau^2} = K_{31}(u_\alpha) + K_{32}(u_z) + K_{33}(w) + p \qquad (3)$$

$$K_{11}(u_\alpha) = \frac{1}{k^2} u_{\alpha,\alpha\alpha} + \frac{2C_2}{k} u_{\alpha,\alpha z} + \left(\frac{C_5}{12k^2} + C_5\right) u_{\alpha,zz} \qquad (4)$$

$$K_{12}(u_z) = \frac{C_2}{k^2} u_{z,\alpha\alpha} + \left(\frac{C_1}{k} + \frac{C_5}{k} - \frac{C_5}{12k^3}\right) u_{z,\alpha z} + + C_4 u_{z,zz} \qquad (5)$$

$$K_{13}(w) = \frac{1}{k^2} w_{,\alpha} - \frac{C_5}{6k^2} w_{,\alpha zz} - \frac{C_2}{12k^3} w_{,\alpha\alpha z} + + \left(\frac{C_2}{k} - \frac{C_2}{12k^3}\right) w_{,z} - \frac{C_4}{12k} w_{,zzz} \qquad (6)$$

$$K_{21}(u_\alpha) = K_{12}(u_\alpha) \qquad (7)$$

$$K_{22}(u_z) = \left(\frac{C_5}{k^2} + \frac{C_5}{12k^4}\right) u_{z,\alpha\alpha} + \frac{2C_4}{k} u_{z,\alpha z} + C_3 u_{z,zz} \qquad (8)$$

$$K_{23}(w) = \frac{C_2}{12k^4} w_{,\alpha\alpha\alpha} + \left(\frac{C_2}{k^2} + \frac{C_2}{12k^4}\right) w_{,\alpha} + \frac{C_5}{6k^3} w_{,\alpha\alpha z} + + \frac{C_4}{12k^2} w_{,\alpha zz} + C_1 w_{,z} \qquad (9)$$

$$K_{31}(u_\alpha) = -K_{13}(u_\alpha) \qquad (10)$$

$$K_{32}(u_z) = -K_{23}(u_z) \qquad (11)$$

$$K_{33}(w) = -\frac{1}{12k^4} w_{,\alpha\alpha\alpha\alpha} - \frac{1}{6k^4} w_{,\alpha\alpha} - \frac{C_1}{6k^2} w_{,\alpha\alpha zz} - - \frac{C_4}{3k} w_{,\alpha zzz} - \frac{C_2}{3k^3} w_{,\alpha\alpha\alpha z} - \frac{C_5}{3k^2} w_{,\alpha\alpha zz} - - \frac{C_2}{3k^3} w_{,\alpha z} - \frac{C_3}{12} w_{,zzzz} - \frac{C_1}{6k^2} w_{,zz} - - \left(\frac{1}{k^2} + \frac{1}{12k^4}\right) w \qquad (12)$$

Here, all variables are dimensionless and the following designations are adopted for them (accents indicate dimensional quantities) (Eqs. 13-28):

$$u_\alpha = \frac{u'_\alpha}{L} \tag{13}$$

$$u_z = \frac{u'_z}{L} \tag{14}$$

$$w = \frac{w'}{L} \tag{15}$$

$$z = \frac{z'}{L} \tag{16}$$

$$c_*^2 = \frac{c_{11}}{\rho} \tag{17}$$

$$\tau = \frac{c_* t}{L} \tag{18}$$

$$q_\alpha = \frac{q'_\alpha L}{h c_{11}} \tag{19}$$

$$q_z = \frac{q'_z L}{h c_{11}} \tag{20}$$

$$p = \frac{P' L}{h c_{11}} \tag{21}$$

$$R = kL \tag{22}$$

$$L = h \tag{23}$$

$$C_1 = \frac{c_{12}}{c_{11}} \tag{24}$$

$$C_2 = \frac{c_{16}}{c_{11}} \tag{25}$$

$$C_3 = \frac{c_{22}}{c_{11}} \tag{26}$$

$$C_4 = \frac{c_{26}}{c_{11}} \tag{27}$$

$$C_5 = \frac{c_{66}}{c_{11}} \tag{28}$$

where u_i' – components of the vector of tangential displacements, w' – normal displacement, c_* – characteristic speed, τ – dimensionless time, t – dimensional time, k – coefficient of the ratio of the shell radius to its thickness, q_i – tangential pressure, P' – normal pressure, L – characteristic dimension.

Equations (1-12) together with the initial conditions (Eqs. 29-34) form the initial problem:

$$u_\alpha|_{\tau=0} = 0 \tag{29}$$

$$\left.\frac{\partial u_\alpha}{\partial \tau}\right|_{\tau=0} = 0 \tag{30}$$

$$u_z|_{\tau=0} = 0 \tag{31}$$

$$\left.\frac{\partial u_\alpha}{\partial \tau}\right|_{\tau=0} = 0 \tag{32}$$

$$w|_{\tau=0} = 0 \tag{33}$$

$$\left.\frac{\partial w}{\partial \tau}\right|_{\tau=0} = 0 \tag{34}$$

RESULTS AND DISCUSSION

Let us denote by $G_w(\alpha,z,\tau)$ the influence function for normal displacement, and by $G_{U\alpha}(\alpha,z,\tau)$ and $G_{Uz}(\alpha,z,\tau)$ the influence functions for tangential displacements. Tangential pressures q_i in (Eqs. 1-3) are assumed to be zero. The influence functions $G_w(\alpha,z,\tau)$, $G_{U\alpha}(\alpha,z,\tau)$ and $G_{Uz}(\alpha,z,\tau)$ – are the solution to the following problem (Eqs. 35-38):

$$\frac{\partial^2 G_{u\alpha}}{\partial \tau^2} = K_{11}\left(G_{u_\alpha}\right) + K_{12}\left(G_{u_z}\right) + K_{13}\left(G_w\right) \tag{35}$$

$$\frac{\partial^2 G_{uz}}{\partial \tau^2} = K_{21}\left(G_{u_\alpha}\right) + K_{22}\left(G_{u_z}\right) + K_{23}\left(G_w\right) \tag{36}$$

$$\frac{\partial^2 G_w}{\partial \tau^2} = K_{31}\left(G_{u_\alpha}\right) + K_{32}\left(G_{u_z}\right) + K_{33}\left(G_w\right) + \\ + \delta(\alpha,z)\delta(\tau) \tag{37}$$

$$G_{u_\alpha}\big|_{\tau=0} = \frac{\partial}{\partial \tau} G_{u_\alpha}\bigg|_{\tau=0} = G_{u_z}\big|_{\tau=0} = \frac{\partial}{\partial \tau} G_{u_z}\bigg|_{\tau=0} = \\ = G_w\big|_{\tau=0} = \frac{\partial}{\partial \tau} G_w\bigg|_{\tau=0} = 0 \tag{38}$$

where $\delta(_*)$ – the Dirac delta function, and the differential operators $K_{ij}\,(G_k)$ have the form (4-12), where it is necessary to replace the corresponding required functions with the influence functions.

To solve this initial problem, we apply to (35-38) expansions in exponential Fourier series in angle α, as well as integral Fourier transforms in the z coordinate and Laplace in time τ. We represent the influence functions and normal pressure in the form of exponential Fourier series (Eqs. 39-43):

$$G_w(\alpha,z,\tau) = \sum_{n=-\infty}^{\infty} G_{wn}(\tau,z) \cdot e^{in\alpha} \tag{39}$$

$$G_{u_\alpha}(\tau,\alpha,z) = \sum_{n=-\infty}^{\infty} G_{u_\alpha n}(\tau,z) \cdot e^{in\alpha} \tag{40}$$

$$G_{u_z}(\alpha,z,\tau) = \sum_{n=-\infty}^{\infty} G_{u_z n}(\tau,z) \cdot e^{in\alpha} \tag{41}$$

$$p(\alpha,z,\tau) = \sum_{n=-\infty}^{\infty} p_n(\tau,z) \cdot e^{in\alpha} \tag{42}$$

$$p_n(\tau,z) = \frac{1}{2\pi}\delta(z)\delta(\tau) \tag{43}$$

Applying to (35-38) the Laplace integral transform in time τ and Fourier transform in the z coordinate, taking into account the properties of integral transformations of the delta function [1], we obtain a system of algebraic equations for the transform images $G_{u_\alpha n}^{LF}, G_{u_z n}^{LF}, G_{wn}^{LF}$, of influence functions in the space of the Fourier and Laplace transforms in the coefficients of the series (Eqs. 44-46):

$$s^2 G_{u_\alpha n}^{LF} - Q_1 G_{u_\alpha n}^{LF} - Q_2 G_{zn}^{LF} - Q_3 G_{wn}^{LF} = 0 \quad (44)$$

$$s^2 G_{u_z n}^{LF} - Q_2 G_{u_\alpha n}^{LF} - Q_4 G_{zn}^{LF} - Q_5 G_{wn}^{LF} = 0 \quad (45)$$

$$s^2 G_{wn}^{LF} + Q_3 G_{u_\alpha n}^{LF} + Q_5 G_{zn}^{LF} - Q_6 G_{wn}^{LF} - \frac{1}{2\pi} = 0 \quad (46)$$

where (Eqs. 47-52):

$$Q_1 = Q_{1n}(q) = -\frac{12C_5k^2q^2 + 24C_2knq + 12n^2 + C_5q^2}{12k^2} \quad (47)$$

$$Q_2 = Q_{2n}(q) = \frac{-12C_4k^3q^2 - 12C_1k^2nq - 12C_5k^2nq - 12C_2kn^2 + C_5nq}{12k^3} \quad (48)$$

$$Q_3 = Q_{3n}(q) = \frac{I\left(C_4k^2q^3 + 2C_5knq^2 + 12C_2k^2q + C_2n^2q + 12kn - C_2q\right)}{12k^3} \quad (49)$$

$$Q_4 = Q_{4n}(q) = -\frac{12C_3k^4q^2 + 24C_4k^3nq + 12C_5k^2n^2 + C_5n^2}{12k^4} \quad (50)$$

$$Q_5 = Q_{5n}(q) = \frac{I\left(-C_4k^2nq^2 - 2C_5kn^2q + 12C_1k^3q - C_2n^3 + 12C_2k^2n + C_2n\right)}{12k^4} \quad (51)$$

$$Q_6 = Q_{6n}(q) = \frac{1}{12k^4}\left(-C_3k^4q^4 - 4C_4k^3nq^3 - 2C_1k^2n^2q^2 - 4C_5k^2n^2q^2 - 4C_2kn^3q - \right.$$
$$\left. -n^4 + 2C_1k^2q^2 + 4C_2knq - 12k^2 + 2n^2 - 1\right) \quad (52)$$

Hereinafter, the superscript "*L*" of the function represents Laplace transform, and "*F*" is its Fourier transform, *s* is the Laplace transform parameter, *q* is the Fourier transform parameter, *n* is the coefficient of the series. Solving system (44-46), we obtain images of the influence function $G_{u_\alpha n}^{LF}, G_{u_z n}^{LF}, G_{wn}^{LF}$. The transform image for the influence function G_{wn}^{LF} will have the form (Eq. 53):

$$G_{wn}^{LF}(q,s) = \frac{-s^4 + s^2R_1 + R_2}{2\pi\left(-s^6 + s^4R_3 + s^2R_4 + R_5\right)} \quad (53)$$

where (Eqs. 54-58):

$$R_1 = R_{1n}(q) = Q_1 + Q_4 \quad (54)$$

$$R_2 = R_{2n}(q) = -Q_1Q_4 + Q_2{}^2 \quad (55)$$

$$R_3 = R_3(q) = Q_1 + Q_4 + Q_6 \quad (56)$$

$$R_4 = R_{4n}(q) = -Q_1Q_4 - Q_1Q_6 + Q_2{}^2 - Q_3{}^2 - Q_4Q_6 - Q_5{}^2 \quad (57)$$

$$R_5 = R_{5n}(q) = Q_1Q_4Q_6 + Q_1Q_5{}^2 - Q_6Q_2{}^2 - 2Q_2Q_3Q_5 + Q_4Q_3{}^2 \quad (58)$$

Let us find the original of the influence function (53). First, we perform the inverse integral Laplace transform. To do this, we split expression (53) into terms by the method of undefined coefficients (Eq. 59):

$$G_{wn}^{LF}(q,s) = \frac{1}{2\pi}\left[\frac{A}{s-s_1} - \frac{A}{s+s_1} + \frac{B}{s-s_2} - \frac{B}{s+s_2} + \frac{C}{s-s_3} - \frac{C}{s+s_3}\right] \quad (59)$$

where (Eqs. 60-62):

$$A = \frac{-s_1{}^4 + Q_{1n}(q)s_1{}^2 + Q_{2n}(q)}{2s_1\left(s_1{}^2 - s_3{}^2\right)\left(s_1{}^2 - s_2{}^2\right)} \quad (60)$$

$$B = \frac{-s_2{}^4 + Q_{1n}(q)s_2{}^2 + Q_{2n}(q)}{2s_2\left(s_1{}^2 - s_2{}^2\right)\left(s_2{}^2 - s_3{}^2\right)} \quad (61)$$

$$C = \frac{-s_3{}^4 + Q_{1n}(q) s_3{}^2 + Q_{2n}(q)}{2 s_3 (s_2{}^2 - s_3{}^2)(s_1{}^2 - s_3{}^2)} \quad (62)$$

and s_i – roots of (Eqs. 53, 63-66):

$$s_1 = \frac{\sqrt{6}\sqrt{U^{\frac{1}{3}}\left(U^{\frac{2}{3}} + 2R_3 U^{\frac{1}{3}} + 4R_3{}^2 + 12R_4\right)}}{6U^{\frac{1}{3}}} \quad (63)$$

$$s_2 = \frac{\sqrt{3U^{\frac{1}{3}}\left(i\sqrt{3}U^{\frac{2}{3}} - 4i\sqrt{3}R_3{}^2 - 12i\sqrt{3}R_4 - U^{\frac{2}{3}} + 4R_3 U^{\frac{1}{3}} - 4R_3{}^2 - 12R_4\right)}}{6U^{\frac{1}{3}}} \quad (64)$$

$$s_3 = \frac{\sqrt{-3U^{1/3}\left(i\sqrt{3}U^{2/3} - 4i\sqrt{3}R_3{}^2 - 12i\sqrt{3}R_4 + U^{2/3} - 4R_3 U^{1/3} + 4R_3{}^2 + 12R_4\right)}}{6U^{1/3}} \quad (65)$$

$$U = 36R_4R_3 + 108R_5 + 8R_3{}^3 + 12\sqrt{12R_5R_3{}^3 - 3R_4{}^2R_3{}^2 + 54R_4R_3R_5 - 12R_4{}^3 + 81R_5{}^2} \quad (66)$$

We use tables [1] and perform the inverse integral Laplace transform of the relation (59) (Eq. 67):

$$G_{wn}^F(q,\tau) = \frac{1}{\pi}\left(A\,sinh(s_1\tau) + B\,sinh(s_2\tau) + C\,sinh(s_3\tau)\right) \quad (67)$$

The Fourier original of the influence function (67) is determined by the well-known inversion formula [1] (Eq. 68):

$$G_{wn}(z,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_{wn}^F(q,\tau) e^{iqz} dq \quad (68)$$

Let us take some large value A and replace improper integral (68) by the definite integral (Eq. 69):

$$G_{wn}(z,\tau) \approx \frac{1}{2\pi}\int_{-A}^{A} G_{wn}^F(q,\tau) e^{iqz} dq \quad (69)$$

To calculate integral (69), we use the numerical method of integrating rapidly oscillating functions [21]. Then the original of the influence function in the coefficients of the series will take the form (Eq. 70):

$$G_{wn}(z,\tau) = \int_{-A}^{A} G_{wn}^F(q,\tau) e^{iqz} dq = \sum_{k=0}^{N-1} \frac{\Delta}{2}\left\{ e^{i\frac{q_{k+1}z + q_k z}{2}} \cdot \left[D_1 G_{wn}^F(q_k,\tau) + D_2 G_{wn}^F(q_{k+1},\tau)\right]\right\} \quad (70)$$

where (Eqs. 71-76):

$$\Delta = \frac{2A}{N} \quad (71)$$

$$m = \frac{\Delta}{2} \quad (72)$$

$$D_{1,2} = \frac{\sin m}{m} \pm \frac{m\cos m - \sin m}{m^2} i \quad (73)$$

$$q_k = A + k\Delta \quad (74)$$

$$q_{k+1} = A + (k+1)\Delta \quad (75)$$

$$k = 0..N-1 \quad (76)$$

Taking into account relations (39-43, 59), and (70), the nonstationary influence function for the normal deflection of an anisotropic unbounded cylindrical Kirchhoff-Love shell takes the form (Eq. 77):

$$G_w(\alpha,z,\tau) = \sum_{n=-\infty}^{\infty}\sum_{k=0}^{N-1}\frac{\Delta}{2}\left\{e^{i\frac{q_{k+1}z + q_k z}{2}}\left[D_1 G_{wn}^F(q_k,\tau) + D_2 G_{wn}^F(q_{k+1},\tau)\right]\right\} e^{in\alpha} \quad (77)$$

Figure 2 shows the graphs of the influence functions $G_{w1}(0,z,1)$ and $G_{w2}(0,z,1)$ depending on the z coordinate at the integration step Δ=0.16 and retention of 10 members of the row. The solid line corresponds to the influence function $G_{w1}(0,z,1)$ at A=10, and the points correspond to the influence function $G_{w2}(0,z,1)$ at A=102. The influence function is plotted with the following dimensionless coefficients characterising the anisotropic material and shell dimensions: C_1=0.814; C_2=-0.735; C_3=0.717; C_4=-0.630; C_5=0.574; k=25. Line – $G_{w1}(0,z,1)$, A=10; Points – $G_{w2}(0,z,1)$, A=102.

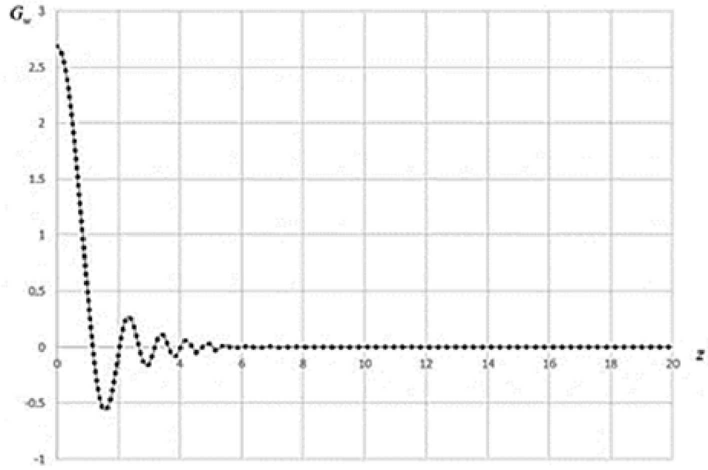
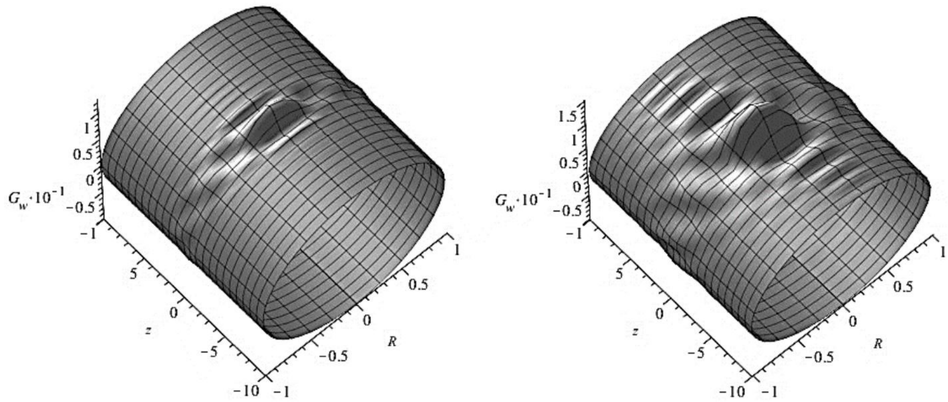


Figure 2: Estimating the convergence of the result

The convergence estimate is performed according to the Chebyshev norm in the space C_0[0.20] (Eq. 78):

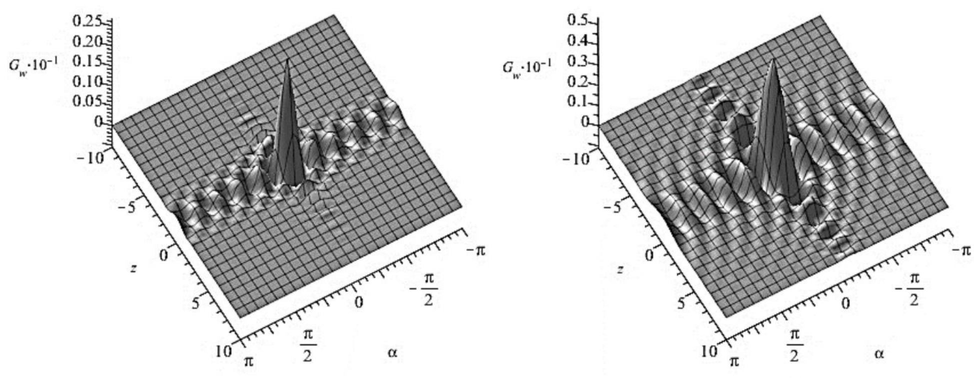
$$\|G_{w1}(0,z,1) - G_{w2}(0,z,1)\| = max_{0\le z\le 20}|G_{w1}(0,z,1) - G_{w2}(0,z,1)| = 0.020218 \quad (78)$$

Figures 3 and 4 show the spatial distributions of the influence function at times τ=1 and τ=4 at *A*=10, *n*=10.



a) τ=1 b) τ=4

Figure 3: Spatial distributions of the influence function



a) τ=1 b) τ=4

Figure 4: Spatial distribution of the influence function in "expanded" form

Results presented in Fig. 3 and Fig. 4 clearly show the effect of material anisotropy on the displacement distribution. The obtained solution demonstrates the asymmetric dynamics of oscillations propagation.

CONCLUSIONS

The process of the impact of a forced non-stationary load on a thin unbounded cylindrical shell of constant thickness is considered. The theory of Kirchhoff-Love plates was accepted as the theory of thin elastic shells. The material of the cylindrical shell is assumed to be elastic and anisotropic. In this case, the case of anisotropy was considered, in which the elastic medium has one plane of symmetry. In this case, this plane was the middle surface of the shell. For a thin Kirchhoff-Love shell, the material under consideration is characterised by six independent elastic constants. The formulation of the problem included the equations of motion of the Kirchhoff-Love elastic shell, the corresponding geometric and physical relations, taking into account the properties of the anisotropy of the shell material under study.

As a result of applying this approach, a non-stationary influence function was found for an anisotropic elastic thin unbounded cylindrical Kirchhoff-Love shell. To find the influence function, the expansion into exponential Fourier series in the angular coordinate, as well as direct and inverse integral Laplace transformation in time and Fourier transformation in the axial coordinate were applied to the original differential equations. The use of the method of undetermined coefficients made it possible to analytically perform the inverse Laplace transformation. The original integral Fourier transform was found by a numerical method for integrating rapidly oscillating functions. The convergence of the result in terms of the Chebyshev norm is estimated when choosing parameters at the stage of transition to a numerical method for integrating rapidly oscillating functions.

The presented results demonstrated the influence of material anisotropy on the distribution of normal displacements – asymmetric dynamics of vibration propagation, which made it possible to assess the adequacy of the found non-stationary influence function. Further research is to investigate the normal deflection of a cylindrical shell in response to the action of a non-stationary load, concentrated or distributed over an arbitrary region and depending arbitrarily on time.

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